Exercise 2

Find the derivative of the function

$$F(x) = \frac{x^4 - 5x^3 + \sqrt{x}}{x^2}$$

in two ways: by using the Quotient Rule and by simplifying first. Show that your answers are equivalent. Which method do you prefer?

Solution

Use the quotient rule to differentiate $F(x)$.

$$F'(x) = \frac{\left[\frac{d}{dx}(x^4 - 5x^3 + \sqrt{x})\right](x^2) - \left[\frac{d}{dx}(x^2)\right](x^4 - 5x^3 + \sqrt{x})}{(x^2)^2}$$

$$= \frac{\left(4x^3 - 15x^2 + \frac{1}{2}x^{-1/2}\right)(x^2) - (2x)(x^4 - 5x^3 + \sqrt{x})}{x^4}$$

$$= \frac{\left(4x^5 - 15x^4 + \frac{1}{2}x^{3/2}\right) - (2x^5 - 10x^4 + 2x^{3/2})}{x^4}$$

$$= \frac{2x^5 - 5x^4 - \frac{3}{2}x^{3/2}}{x^4}$$

$$= 2x - 5 - \frac{3}{2}x^{-5/2}$$

Expand the function first

$$F(x) = x^2 - 5x + x^{-3/2}$$

and then differentiate it.

$$F'(x) = \frac{d}{dx}(x^2 - 5x + x^{-3/2})$$

$$= 2x - 5 - \frac{3}{2}x^{-5/2}$$

Both approaches give the same answer, but the second one is less work.